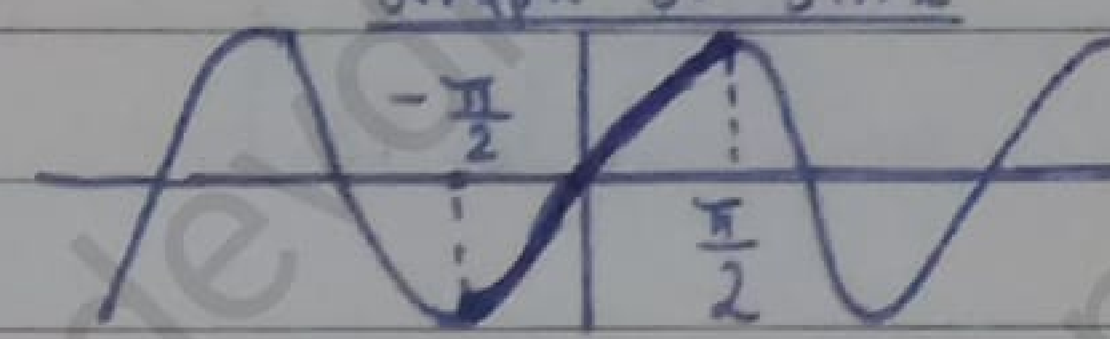
16/5/23

CHAPTER - 2

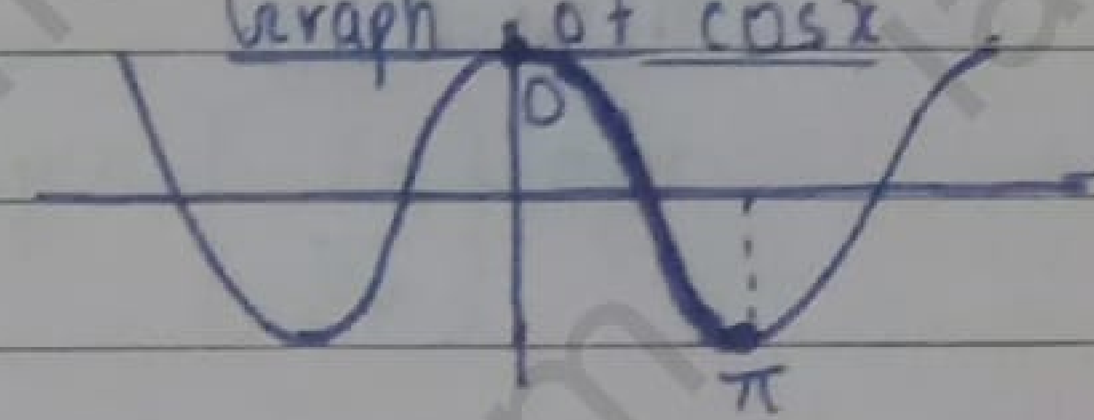
INVERSE TRIGONOMETRIC FUNCTIONS

FUNCTION	DOMAIN	RANGE
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\sec^{-1}x$	$R-(-1,1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
$\text{cosec}^{-1}x$	$R-(-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$

Graph of sin x



Graph of cos x



- $R^{-1} = \{(y,x) : (x,y) \in R\}$

- $\sin^{-1}(\theta) \neq \dfrac{1}{\sin\theta}$

- $\sin^{-1}(-\theta) = -\sin^{-1}\theta$
- $\text{cosec}^{-1}(-\theta) = -(\text{cosec}^{-1}\theta)$
- $\tan^{-1}(-\theta) = -\tan^{-1}\theta$
- $\cot^{-1}(-\theta) = \pi - \cot^{-1}\theta$
- $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$
- $\sec^{-1}(-\theta) = \pi - \sec^{-1}(\theta)$

EXAMPLE 1

Q. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

Sol. $\sin^{-1}\sin\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$ Ans

EXAMPLE 2

Q Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$.

Sol. Method I

$$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \text{ Ans}$$

Method II

$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\cot y = \frac{-1}{\sqrt{3}}$$

$$\cot y = \cot\left(\pi - \frac{\pi}{3}\right) = \cot\frac{2\pi}{3}$$

$$y = \frac{2\pi}{3} \text{ Ans}$$

Q Find principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

Sol. $$\sin^{-1}\left(-\sin\frac{\pi}{3}\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3} \text{ Ans}$$

- TRIGONOMETRIC FORMULAS

* $$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

* $$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

* $1 + \cos 2\theta = 2\cos^2\theta$ $\qquad$ $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

$1 - \cos 2\theta = 2\sin^2\theta$ $\qquad$ $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

* $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

- $a^2+x^2 \Rightarrow x = a\tan\theta$ OR $a\cot\theta$
- $1+x^2 \Rightarrow x = \tan\theta$ OR $\cot\theta$

- $a^2-x^2 \Rightarrow x = a\sin\theta$ OR $a\cos\theta$
- $1-x^2 \Rightarrow x = \sin\theta$ OR $\cos\theta$

- $x^2-a^2 \Rightarrow x = a\sec\theta$ OR $a\operatorname{cosec}\theta$
- $x^2-1 \Rightarrow x = a\sec\theta$ OR $a\operatorname{cosec}\theta$

- $\tan^{-1}x + \tan^{-1}y = \cancel{\tan^{-1}\frac{x+y}{1-xy}}\ \tan^{-1}\frac{x+y}{1-xy}$
- $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$
- $2\tan^{-1}x = \frac{2x}{1-x^2}$
- $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

16/5/23

★ EXERCISE - 2.1

- Find the principal values of the following:

Q1.) $\sin^{-1}\left(-\frac{1}{2}\right)$

Sol. $\sin^{-1}\left(-\sin\frac{\pi}{6}\right) = \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$ Ans

Q2.) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Sol. $\cos^{-1}\cos\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$ Ans

Q3.) $\operatorname{cosec}^{-1}(2)$

Sol. $\operatorname{cosec}^{-1}\operatorname{cosec}\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$ Ans

Q4.) $\tan^{-1}(-\sqrt{3})$

Sol. $\tan^{-1}\left(-\tan\left(\frac{\pi}{3}\right)\right) = \tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$ Ans

Q5.) $\cos^{-1}\left(-\frac{1}{2}\right)$

Sol. $\cos^{-1}\left[-\frac{1}{2}\right] = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \cos^{-1}\cos\left(\frac{\pi}{3}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q6.) $\tan^{-1}(-1)$

Sol. $\tan^{-1}(-1) = -\tan^{-1}(1) = -\tan^{-1}\tan\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}$ Ans

Q7.) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Sol. $\sec^{-1}\sec\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$ Ans

Q8.) $\cot^{-1}(\sqrt{3})$

Sol. $\cot^{-1}\cot\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$ Ans

Q9.) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Sol. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ Ans

Q10.) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Sol. $\operatorname{cosec}^{-1}(-\sqrt{2}) = -\operatorname{cosec}^{-1}(\sqrt{2}) = -\operatorname{cosec}^{-1}\operatorname{cosec}\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}$ Ans

• Find the values of the following:

Q11.) $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

Sol. $\left[\tan^{-1}\tan\left(\frac{\pi}{4}\right)\right] + \left[\pi - \cos^{-1}\cos\left(\frac{\pi}{3}\right)\right] + \left[-\sin^{-1}\left(\frac{\pi}{6}\right)\right]$

$\Rightarrow \cancel{\frac{\pi}{4} + \tan^{-1}\tan\left(\frac{\pi}{4}\right)}\ \frac{\pi}{4} + \pi - \frac{\pi}{3} - \frac{\pi}{6}$

$\Rightarrow \frac{3\pi + 12\pi - 4\pi - 2\pi}{12}$

$\Rightarrow \frac{9\pi}{12}$

$\Rightarrow \frac{3\pi}{4}$ Ans

Q12.) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Sol. $\cos \cos^{-1}\left(\frac{\pi}{3}\right) + 2\sin^{-1}\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow \frac{\pi}{3} + \frac{\cancel{2}\pi}{\cancel{6}3}$

$\Rightarrow \frac{2\pi}{3}$ Ans

Q13.) If $\sin^{-1}x = y$, then

(A) $0 \le y \le \pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Q14.) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is equal to

(A) π (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Sol. $\tan^{-1}\tan\left(\frac{\pi}{3}\right) - \pi + \cancel{\sin}\ \sec^{-1}\sec\left(\frac{\pi}{6}\right) = \frac{\pi}{3} - \pi + \frac{\pi}{6} = \frac{-2\pi}{6} = -\frac{\pi}{3}$ Ans (B)

17/5/23

☆ EXERCISE - 2.2

- Prove the following:

Q1.) $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Sol. ~~R.H.S.~~

~~$\sin^{-1}(3x-4x^3)$~~

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

R.H.S.

$\sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$\Rightarrow \sin^{-1}\sin(3\theta)$

$\Rightarrow 3\theta$

$\Rightarrow 3\sin^{-1}x = L.H.S$

Hence, proved

Q2.) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$

Sol. ~~R.H.S~~

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

R.H.S.

$\cos^{-1}(4x^3 - 3x)$

$\Rightarrow \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$\Rightarrow \cos^{-1}\cos(3\theta)$

$\Rightarrow 3\theta$

$\Rightarrow 3\cos^{-1}x = L.H.S.$

Hence, proved

- Write the following functions in the simplest form:

Q3.) $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$

Sol. Let $x = \tan\theta$

$\tan^{-1}\dfrac{(\sqrt{1+\tan^2\theta}-1)}{\tan\theta}$

$\Rightarrow \tan^{-1}\dfrac{\sec\theta-1}{\dfrac{\sin\theta}{\cos\theta}}$

$\Rightarrow \tan^{-1}\dfrac{\dfrac{1}{\cos\theta}-1}{\dfrac{\sin\theta}{\cos\theta}}$

$\Rightarrow \tan^{-1}\dfrac{(1-\cos\theta)\cancel{\cos\theta}}{\cancel{\cos\theta}\times\sin\theta}$

$\Rightarrow \tan^{-1}\dfrac{\cancel{2}\sin^{\cancel{2}}\theta/2}{\cancel{2}\cancel{\sin\theta/2}\cos\theta/2}$

$\Rightarrow \tan^{-1}\tan\dfrac{\theta}{2}$

$\Rightarrow \dfrac{\theta}{2}$

$\Rightarrow \dfrac{1}{2}\tan^{-1}x$ Ans

Q4.) $\tan^{-1}\left(\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)$, ~~$x<\pi$~~ $0<x<\pi$

Sol. $\tan^{-1}\sqrt{\dfrac{\cancel{2}\sin^2 x/2}{\cancel{2}\cos^2 x/2}}$ $\quad 0<\dfrac{x}{2}<\dfrac{\pi}{2}$

$\Rightarrow \tan^{-1}\sqrt{\tan^2 x/2}$ $\quad 0<\dfrac{x}{2}<\dfrac{\pi}{2}$

$\Rightarrow \tan^{-1}\tan\dfrac{x}{2}$

$\Rightarrow \dfrac{x}{2}$ Ans

Q5.) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$

Sol. $\tan^{-1}\left(\dfrac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\tan \pi/4 - \tan x}{1 + \tan \pi/4 \tan x}\right)$

$\Rightarrow \tan^{-1} \tan\left(\frac{\pi}{4} - x\right)$

$\Rightarrow \frac{\pi}{4} - x$ Ans

Q6.) $\tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$, $|x| < a$

Sol. $\tan^{-1} \frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$ [Let $x = a\sin\theta$]

$\Rightarrow \tan^{-1} \frac{a\sin\theta}{\sqrt{a^2(1 - \sin^2\theta)}}$

$\Rightarrow \tan^{-1} \frac{\cancel{a}\sin\theta}{\cancel{a}\sqrt{1 - \sin^2\theta}}$

$\Rightarrow \tan^{-1} \frac{\sin\theta}{\sqrt{\cos^2\theta}}$

$\Rightarrow \tan^{-1} \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan^{-1} \tan\theta$

$\Rightarrow \theta$

$x = a\sin\theta$

$\theta = \sin^{-1} \frac{x}{a}$ Ans

Q7.) $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$, $a > 0$; $\frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$

Sol. Let $x = a\tan\theta \Rightarrow \theta = \tan^{-1}\frac{x}{a}$

$\tan^{-1}\left(\frac{3a^2 a\tan\theta - a^3\tan^3\theta}{a^3 - 3a\,a^2\tan^2\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\cancel{a^3}(3\tan\theta - \tan^3\theta)}{\cancel{a^3}(1 - \tan^2\theta)}\right)$

$\Rightarrow \tan^{-1} \tan(3\theta)$

$\Rightarrow 3\theta$

$\Rightarrow 3\tan^{-1}\frac{x}{a}$ Ans

- Find the values of each of the expression in Exercises 10 to 15.

Q 10.) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Sol. $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right]$

$\Rightarrow \sin^{-1}\sin\left(\frac{\pi}{3}\right)$

$\Rightarrow \frac{\pi}{3}$ Ans

Q 11.) $\tan^{-1}\left[\tan\frac{3\pi}{4}\right]$

Sol. $\frac{3\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}\tan\left(\pi - \frac{\pi}{4}\right)$

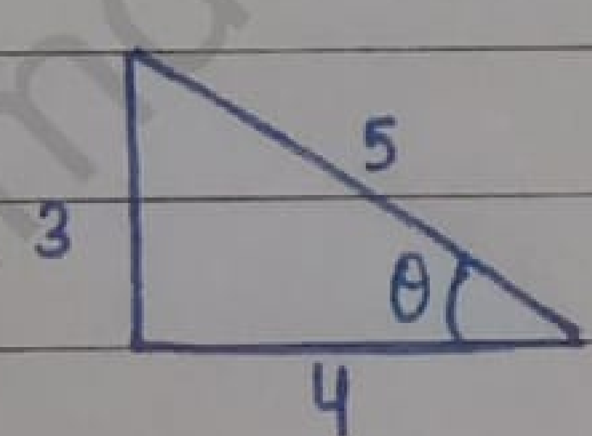
$\Rightarrow \tan^{-1}\tan\left(-\frac{\pi}{4}\right)$

$\Rightarrow -\frac{\pi}{4}$ Ans

Q 12.) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

Sol. Let $\sin^{-1}\frac{3}{5} = \theta$

$\Rightarrow \sin\theta = \frac{3}{5}$



$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{3}{4}$

$\tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{\cancel{3}}{4}\times\frac{2}{\cancel{3}}}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\frac{9+8}{12}}{\frac{1}{2}}\right)$

$\Rightarrow \tan \tan^{-1} \frac{34}{12}$

$\Rightarrow \frac{34}{12}$

$\Rightarrow \frac{17}{6}$ Ans

Q 13.) $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ is equal to (A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Sol. $\frac{7\pi}{6} \notin [0, \pi]$

$\cos^{-1} \cos\left(2\pi - \frac{5\pi}{6}\right)$

$\Rightarrow \cos^{-1} \cos \frac{5\pi}{6}, \quad \frac{5\pi}{6} \in [0, \pi]$

$\Rightarrow$ (D) $\frac{\pi}{6}$ Ans

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h.w.

Q8.) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Sol. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\sin\frac{\pi}{6}\right)\right]$

$\Rightarrow \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$\Rightarrow \tan^{-1} \cancel{2} \times \frac{1}{\cancel{2}}$

$\Rightarrow \tan^{-1} \tan \frac{\pi}{4}$

$\Rightarrow \frac{\pi}{4}$ Ans

Q13) $\tan \frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x|<1$, $y>0$ and $xy<1$

Sol. Let $x = \tan\theta$, $y = \tan\phi$

$$\tan \frac{1}{2}\left[\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta} + \cos^{-1}\frac{1-\tan^2\phi}{1+\tan^2\phi}\right]$$

$$\Rightarrow \cancel{\tan \frac{1}{2}\left[\sin^{-1}\sin 2\theta + \cos^{-1}\cos 2\theta\right]}$$

$$\Rightarrow \tan \frac{1}{2}\left(\sin^{-1}\sin 2\theta + \cos^{-1}\cos 2\phi\right)$$

$$\Rightarrow \tan \frac{1}{2}(2\theta + 2\phi)$$

$$\Rightarrow \tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}$$

$$\Rightarrow \frac{x+y}{1-xy}$$ Ans

Q14) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

Sol. $\sin\left(\frac{\pi}{3} + \sin^{-1}\sin\frac{\pi}{6}\right)$

$$\Rightarrow \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$$

$$\Rightarrow \sin\left(\frac{2\pi + \pi}{6}\right)$$

$$\Rightarrow \sin\frac{3\pi}{6} = \sin\frac{\pi}{2}$$

$\Rightarrow$ (D) 1 Ans

Q15.) $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to

(A) π (B) $-\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

Sol. $\tan^{-1}\tan\frac{\pi}{3} - \pi + \cot^{-1}\cot\frac{\pi}{6}$

$\Rightarrow \frac{\pi}{3} - \pi + \frac{\pi}{6}$

$\Rightarrow \frac{2\pi - 6\pi + \pi}{6}$

$\Rightarrow \frac{-3\pi}{6}$

$\Rightarrow$ (B) $-\frac{\pi}{2}$ Ans

17/5/23

★ MISCELLANEOUS EXERCISE ON CHAPTER 2

- Find the value of the following :

Q1.) $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$

Sol. $\frac{13\pi}{6} \notin [0, \pi]$

$\cos^{-1}\left(\cos\left(\frac{\cancel{12}^{2}\pi}{\cancel{6}} + \frac{\pi}{6}\right)\right)$

$\Rightarrow \cos^{-1}(\cos \frac{\pi}{6}$, $\frac{\pi}{6} \in [0, \pi]$

$\Rightarrow \frac{\pi}{6}$ Ans

Q2.) $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$

Sol. $\frac{7\pi}{6} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}\left(\tan\left(\frac{\cancel{6}\pi}{\cancel{6}} + \frac{\pi}{6}\right)\right)$

$\Rightarrow \tan^{-1} \tan \frac{\pi}{6}$

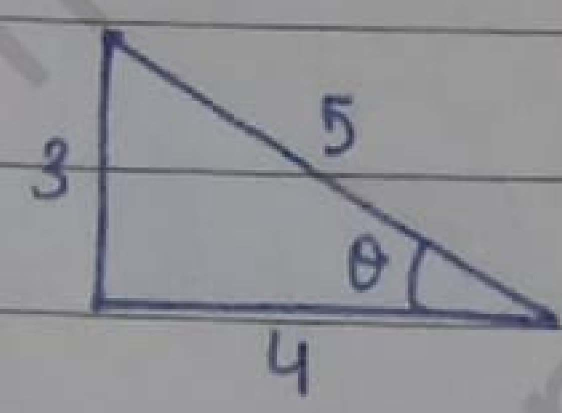
$\Rightarrow \frac{\pi}{6}$ Ans

- Prove that

Q3.) $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Sol. Let $\sin^{-1}\frac{3}{5} = \theta$

$\Rightarrow \sin\theta = \frac{3}{5}$



$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{3}{4}$

<u>L.H.S.</u>

$2\tan^{-1}\frac{3}{4}$

$\Rightarrow \cancel{\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{4}}\ \tan^{-1}\left(\frac{2\times 3/4}{1 - 9/16}\right)$

$\Rightarrow \cancel{\tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{4}}{1-\frac{3}{4}\times\frac{3}{4}}\right)}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{3}{2}}{1-\frac{9}{16}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{3/2}{7/16}\right)$

$\Rightarrow \tan^{-1}\frac{48}{14}$

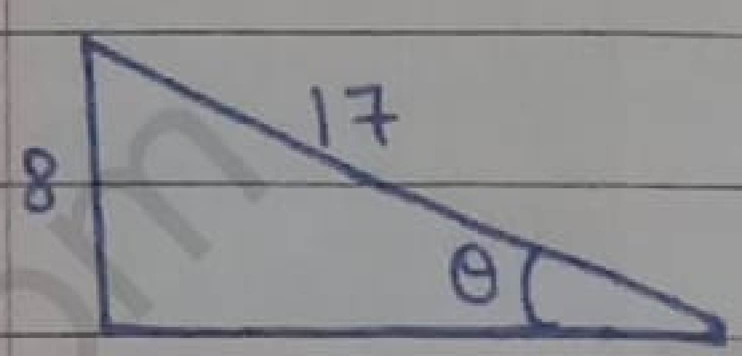
$\Rightarrow \tan^{-1}\frac{24}{7}$

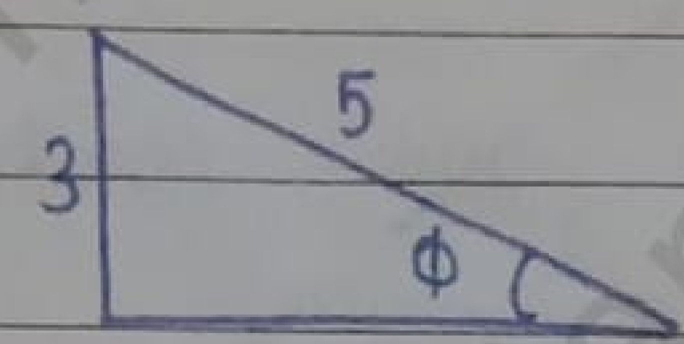
<u>hence, proved</u>

Q4.) $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Sol. Let $\theta = \sin^{-1}\frac{8}{17}$ and $\phi = \sin^{-1}\frac{3}{5}$

$\Rightarrow \sin\theta = \frac{8}{17}$ $\qquad$ $\sin\phi = \frac{3}{5}$





$H^2 = B^2 + P^2$

$\Rightarrow 289 = B^2 + 64$

$\Rightarrow B^2 = 225$

$\Rightarrow B = 15$

$H^2 = B^2 + P^2$

$\Rightarrow 25 = B^2 + 9$

$\Rightarrow B^2 = 16$

$\Rightarrow B = 4$

$\tan\theta = \frac{8}{15}$

$\Rightarrow \theta = \tan^{-1}\frac{8}{15}$

$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{3}{4}$

L.H.S

$\tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right) \Rightarrow \tan^{-1}\left(\frac{\frac{32+45}{60}}{1-\frac{24}{60}}\right) \Rightarrow \tan^{-1}\left(\frac{\frac{77}{60}}{\frac{60-24}{60}}\right)$

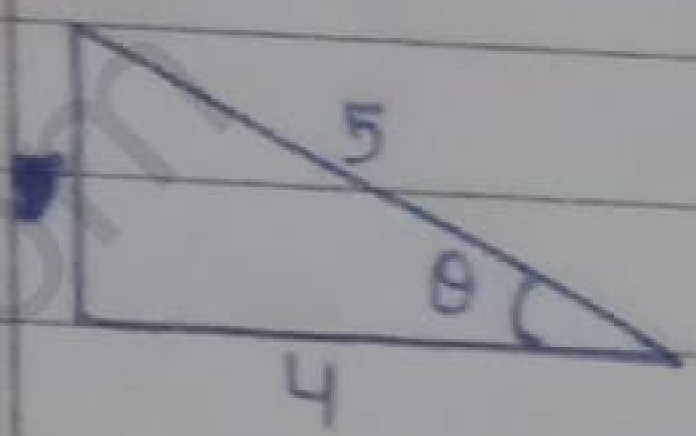
$\Rightarrow \tan^{-1}\frac{77}{36} = R.H.S.$

<u>Hence, proved</u>

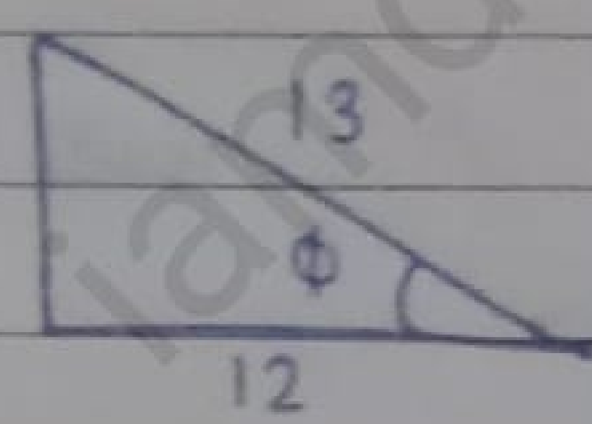
Q5.) $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Sol. Let $\cos^{-1}\frac{4}{5} = \theta$ and $\cos^{-1}\frac{12}{13} = \phi$

$\Rightarrow \cos\theta = \frac{4}{5}$ and $\cos\phi = \frac{12}{13}$



and



$H^2 = B^2 + P^2$

$\Rightarrow 25 = 16 + P^2$

$\Rightarrow P^2 = 9$

$\Rightarrow P = 3$

$H^2 = B^2 + P^2$

$\Rightarrow 169 = 144 + P^2$

$\Rightarrow P^2 = 25$

$\Rightarrow P = \cancel{2}5$

L.H.S

$$\cos^{-1}\left(\frac{4}{5} \times \frac{12}{13} - \sqrt{1 - \frac{16}{25}}\sqrt{1 - \frac{144}{169}}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{48}{65} - \sqrt{\frac{25-16}{25}}\sqrt{\frac{169-144}{169}}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{48}{65} - \sqrt{\frac{9}{25}}\sqrt{\frac{25}{169}}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{48}{65} - \frac{15}{65}\right)$$

$$\Rightarrow \cos^{-1}\frac{33}{65} = \text{R.H.S.}$$

Hence, proved

Q9.) $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, \; x \in \left(0, \frac{\pi}{4}\right)$

Sol. $\sqrt{1+\sin x} = \sqrt{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}$

$$= \sqrt{(\cos x/2 + \sin x/2)^2}$$
$$= \cos x/2 + \sin x/2$$

$$\sqrt{1-\sin x} = \sqrt{\sin^2 x/2 + \cos^2 x/2 - 2\sin x/2 \cos x/2}$$
$$= \sqrt{(\cos x/2 - \sin x/2)^2}$$
$$= \cos x/2 - \sin x/2$$

$$\cot^{-1}\left(\frac{\cos x/2 + \sin x/2 + \cos x/2 - \sin x/2}{\cos x/2 + \sin x/2 - \cos x/2 + \sin x/2}\right)$$
$$\Rightarrow \cot^{-1} \cot \frac{x}{2}$$
$$\Rightarrow \frac{x}{2} = \text{R.H.S.}$$

Hence, proved

Q10.) $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \ -\frac{1}{\sqrt{2}} \leq x \leq 1$

Sol. Let $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$$\sqrt{1+\cos 2\theta} \Rightarrow \sqrt{2\cos^2\theta} \Rightarrow \sqrt{2}\cos\theta$$
$$\sqrt{1-\cos 2\theta} \Rightarrow \sqrt{2\sin^2\theta} \Rightarrow \sqrt{2}\sin\theta$$

L.H.S.

$$\tan^{-1}\left(\frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right)$$
$$\Rightarrow \tan^{-1}\left(\frac{\cancel{\sqrt{2}}(\cos\theta - \sin\theta)}{\cancel{\sqrt{2}}(\cos\theta + \sin\theta)}\right)$$
$$\Rightarrow \tan^{-1}\left(\frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\right)$$

$\Rightarrow \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\tan \pi/4 - \tan\theta}{1+\tan \pi/4 \tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\tan \frac{\pi}{4} - \theta\right)$

$\Rightarrow \frac{\pi}{4} - \theta$

$\Rightarrow \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$ $= R.H.S.$

Hence, proved

- Solve the following equations:

Q11.) $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\, x)$

Sol. $\tan^{-1}\frac{2\cos x}{1-\cos^2 x} = \tan^{-1}(2\,\mathrm{cosec}\, x)$

$\Rightarrow \frac{\cancel{2}\cos x}{\sin^2 x} = \cancel{2}\,\mathrm{cosec}\, x$

$\Rightarrow \frac{\cos x}{\sin^2 x \times \mathrm{cosec}\, x} = 1$

$\Rightarrow \frac{\cos x}{\sin x} = 1$

$\Rightarrow \cot x = 1$

$\Rightarrow x = 45^\circ$ Ans

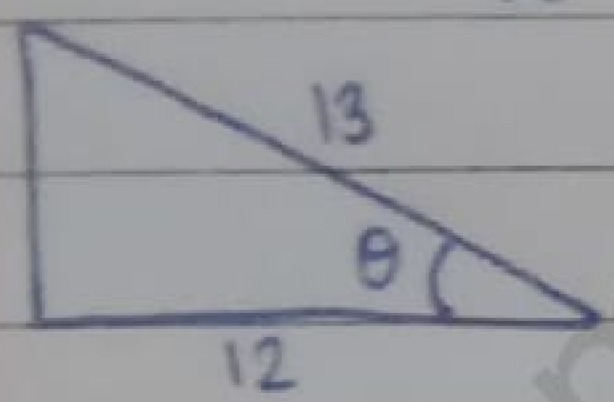
H.W.

Q6.) $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

Sol. ~~Let~~ $\cancel{\sin^{-1}\frac{3}{5} = \theta}$ ~~and~~ $\cancel{\sin^{-1}\frac{56}{65}}$

Let $\cos^{-1}\frac{12}{13} = \theta$

$\Rightarrow \cos\theta = \frac{12}{13}$



$H^2 = B^2 + P^2$

$\Rightarrow 169 = 144 + P^2$

$\Rightarrow P^2 = 25$

$\Rightarrow P = 5$

$\cancel{\tan\theta = \frac{5}{12}}$ $\sin\theta = \frac{5}{13}$

$\Rightarrow \theta = \sin^{-1}\frac{5}{13}$

L.H.S.

$\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{3}{5}$

$\Rightarrow \sin^{-1}\left(\frac{5}{13}\sqrt{1-\frac{9}{25}} + \frac{3}{5}\sqrt{1-\frac{25}{169}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{5}{13}\sqrt{\frac{25-9}{25}} + \frac{3}{5}\sqrt{\frac{169-25}{169}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{5}{13}\sqrt{\frac{16}{25}} + \frac{3}{5}\sqrt{\frac{144}{169}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{\cancel{5}}{13}\times\frac{4}{\cancel{5}} + \frac{3}{5}\times\frac{12}{13}\right)$

$\Rightarrow \sin^{-1}\left(\frac{4}{13} + \frac{36}{65}\right)$

$\Rightarrow \sin^{-1}\left(\frac{20+36}{65}\right)$

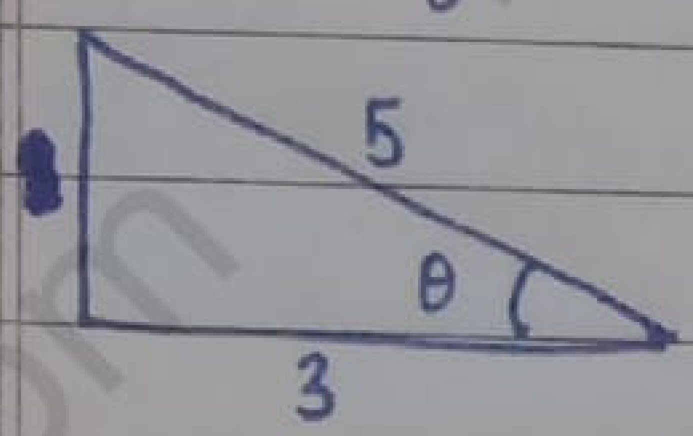
$\Rightarrow \sin^{-1}\frac{56}{65}$ = R.H.S.

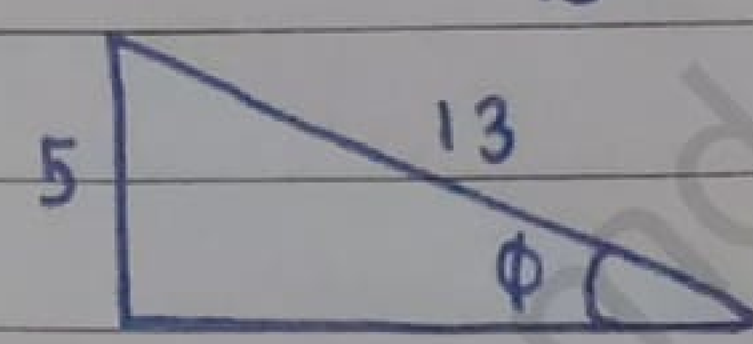
Hence, proved

Q7.) $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Sol. Let $\cos^{-1}\frac{3}{5} = \theta$ and $\sin^{-1}\frac{5}{13} = \phi$

$\Rightarrow \cos\theta = \frac{3}{5}$ $\Rightarrow \sin\phi = \frac{5}{13}$





$h^2 = B^2 + P^2$
$\Rightarrow 25 = 9 + P^2$
$\Rightarrow P^2 = 16$
$\Rightarrow P = 4$

$h^2 = B^2 + P^2$
$\Rightarrow 169 = B^2 + 25$
$\Rightarrow B^2 = 144$
$\Rightarrow B = 12$

$\tan\theta = \frac{4}{3}$
$\Rightarrow \theta = \tan^{-1}\frac{4}{3}$

$\tan\phi = \frac{5}{12}$
$\Rightarrow \phi = \tan^{-1}\frac{5}{12}$

R.H.S.

$\tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right) \Rightarrow \tan^{-1}\left(\frac{\frac{5+16}{12}}{1-\frac{20}{36}}\right) \Rightarrow \tan^{-1}\left(\frac{\frac{21}{12}}{\frac{16}{36}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{21\times \cancel{36}^{3}}{\cancel{12}\times 16}\right)$

$\Rightarrow \tan^{-1}\frac{63}{16} = \text{L.H.S.}$

Hence, Proved

Q8.) $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x}$, $x \in [0,1]$

Sol. Let $x = \tan^2\theta \Rightarrow \sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

R.H.S.

$\frac{1}{2}\cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \frac{1}{2}\cos^{-1}\cos 2\theta$

$\Rightarrow \frac{1}{2}\times 2\theta$

$\Rightarrow \theta$

$\Rightarrow \tan^{-1}\sqrt{x} = $ L.H.S.

Hence, proved

Q12.) $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$, $(x>0)$

Sol. ~~$\tan^{-1}\frac{1-x}{}$~~

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\tan^{-1}\frac{1-\tan\theta}{1+\tan\theta} = \frac{\theta}{2}$

$\Rightarrow \tan^{-1}\left(\frac{\tan\pi/4 - \tan\theta}{1+\tan\pi/4\tan\theta}\right) = \frac{\theta}{2}$

$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\theta}{2}$

$\Rightarrow \frac{\pi}{4} - \theta = \frac{\theta}{2}$

$\Rightarrow \frac{\pi}{2} - 2\theta = \theta$

$\Rightarrow \frac{\pi}{2} = 3\theta$

$\Rightarrow \frac{\pi}{2} = 3\tan^{-1}x$

$\Rightarrow \tan^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$ Ans

Q13.) $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Sol. Let $\theta = \tan^{-1}x \Rightarrow \tan\theta = x$

$\sin(\tan^{-1}\tan\theta)$

$\Rightarrow \sin\theta$

$x = \tan\theta$

$\Rightarrow x = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \sin\theta = x\cos\theta$ —— ①

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec\theta = \sqrt{1+x^2}$

① $\Rightarrow x \times \frac{1}{\sec\theta}$

$\Rightarrow$ (D) $\frac{x}{\sqrt{1+x^2}}$ Ans

$\therefore$ (D)

Q14.) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

Sol. $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$

$\Rightarrow 1-x = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$\Rightarrow 1-x = \cos(2\sin^{-1}x)$ ①

$\cancel{1-x = 1-2x^2}$

Let $\sin^{-1}x = y$

$\Rightarrow \sin y = x$

① $\Rightarrow 1-x = \cos(2\sin^{-1}x)$

$\Rightarrow 1-x = \cos 2y$

$\Rightarrow 1-x = 1-2\sin^2 y$

$\Rightarrow \cancel{1}-x = \cancel{1}-2x^2$

$\Rightarrow 2x^2-x=0$

$\Rightarrow x(2x-1)=0$

$x=0, \; x=\frac{1}{2}$

L.H.S.

$\sin^{-1}(1-0) - 2\sin^{-1}0$

$\Rightarrow \sin^{-1}\sin\frac{\pi}{2} - 2\sin^{-1}\sin 0$

$\Rightarrow \frac{\pi}{2} = \text{R.H.S}$

L.H.S.

$\sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$

$\Rightarrow \sin^{-1}\sin\frac{\pi}{6} - 2\sin^{-1}\sin\frac{\pi}{6}$

$\Rightarrow \frac{\pi}{6} - \frac{\pi}{3}$

$\Rightarrow \frac{\pi - 2\pi}{6}$

$\Rightarrow -\frac{\pi}{6} \neq \text{R.H.S.}$

$\therefore$ (C) 0 Ans